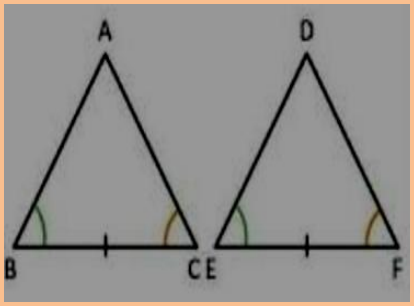
General instructions for Students: Whatever be the notes provided, everything must be copied in the Maths copy and then do the HOMEWORK in the same copy.

SOME MORE CRITERIA FOR CONGRUENCE OF TRIANGLES

ASA congruence rule –

Statement: *Two triangles are congruent if two angles and the included side of a triangle are equal to two angles and the included side of the other triangle.*



Given: In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E$, $\angle C = \angle DFE$ and $BC = EF$

To Prove: $\triangle ABC \cong \triangle DEF$

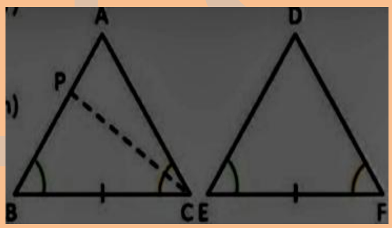
Proof: **Case I** Let $AB = DE$.

In $\triangle ABC$ and $\triangle DEF$,

$AB = DE$, $BC = EF$ and $\angle B = \angle E$

$\therefore$ $\triangle ABC \cong \triangle DEF$ (*SAS congruence rule*)

Case II Let $AB > DE$, Take a point P on AB such that $PB = DE$ and we join PC.



In $\triangle PBC$ and $\triangle DEF$,

$PB = DE$, $\angle B = \angle E$ and $BC = EF$

$\therefore$ $\triangle PBC \cong \triangle DEF$ (*SAS congruence rule*)

$\therefore$ $\angle PCB = \angle DFE$ (*By C.P.C.T.*)

But, $\angle C = \angle DFE$ (*Given*)

$\therefore$ $\angle PCB = \angle ACB$ *This is possible only when A and P coincide.*

$\therefore$ $PB = AB \implies AB = DE$

$\therefore$ $\triangle ABC \cong \triangle DEF$ (*SAS congruence rule*) *Proved.*

Case III Let $AB < DE$, we can take a point M on DE such that $AB = ME$ and join MF.

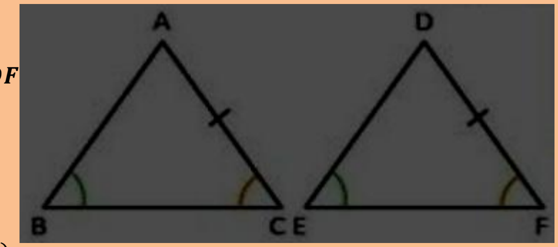
Similarly repeating statements as given in Case II

We can conclude that $AB = DE$

$\therefore$ $\triangle ABC \cong \triangle DEF$ (*SAS congruence rule*) *Proved.*

AAS congruence rule –

***Statement :** Two triangles are congruent if any two pairs of angles and a pair of corresponding sides are equal.*



Given : In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E$, $\angle C = \angle F$ and $AC = DF$

To Prove : $\triangle ABC \cong \triangle DEF$

Proof : $\angle A + \angle B + \angle C = 180°$ ………… (i)

(By Angles sum prop. of a triangle)

$\angle D + \angle E + \angle F = 180°$ ………… (ii) (By Angles sum prop. of a triangle)

From (i) and (ii), $\angle A + \angle B + \angle C = \angle D + \angle E + \angle F$

$\therefore$ $\angle A = \angle D$ ……… (iii) (Given : $\angle B = \angle E$, $\angle C = \angle F$)

In $\triangle ABC$ and $\triangle DEF$, $\angle C = \angle F$ (Given)

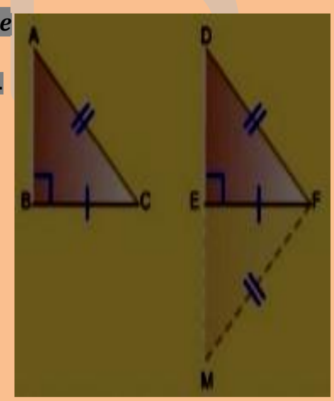
$AC = DF$ (Given)

$\angle A = \angle D$ By (iii)

$\triangle ABC \cong \triangle DEF$ (By ASA congruence rule) ***Proved.***

RHS congruence rule –

***Statement :** Two right triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle.*



Given : In two right – angled triangles $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E = 90°$

$BC = EF$ and $AC = DF$ (hypotenuse)

To Prove : $\triangle ABC \cong \triangle DEF$

Construction : Produced DE to M such that $EM = AB$. We join MF.

Proof : In $\triangle ABC$ and $\triangle MEF$,

$BC = EF$ (Given)

$AB = EM$ (By Const.)

$\angle B = \angle MEF$ $(= 90°)$

$\therefore$ $\triangle ABC \cong \triangle MEF$ (SAS congruence rule)

$\therefore$ $\angle A = \angle M$ and $AC = MF$ (By C.P.C.T.) ……………… (i)

But $AC = DF$ (Given) ……………………… (ii)

From (i) and (ii), $DF = MF$

$\therefore \quad \angle D = \angle M \ldots\ldots\ldots\ldots\ldots\ldots..(iii) \quad (\angle s \text{ opp. to equal sides are equal})$

From (i) **and** (iii), $\quad \angle A = \angle D \ldots\ldots\ldots\ldots\ldots\ldots..(iv)$

Now, $\quad \angle A = \angle D, \ \angle B = \angle E \implies \angle C = \angle EFD \ldots\ldots\ldots\ldots\ldots\ldots\ldots(v)$

Now, $\quad$ In $\triangle ABC$ and $\triangle DEF$,

$BC = EF \qquad (Given)$

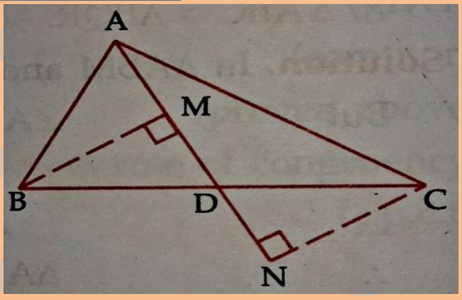
$AC = DF \qquad (Given)$

$\angle C = \angle EFD \qquad By\ (v)$

$\therefore \quad \triangle ABC \cong \triangle DEF \qquad (SAS \text{ congruence rule}) \qquad$ **Proved.**

EXERCSE – 10.2

Q.No.4 $\quad$ *In the adjoining figure, AD is a median of $\triangle ABC$, BM and CN are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that BM = CN.*



Solution: $\quad$ In $\triangle BMD$ and $\triangle CND$,

$\angle BMD = \angle CND \qquad (= 90°)$

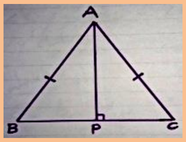
$\angle BDM = \angle CDN \qquad (vert.\ opp.\ \angle s)$

$BD = DC \qquad (AD \text{ is a median})$

$\therefore \quad \triangle BMD \cong \triangle CND \qquad (AAS \text{ congruence rule})$

$\therefore \quad BM = CN \qquad (By\ \ C.P.C.T.) \qquad$ **Proved.**

Q.No.10 $\quad$ *ABC is an isosceles triangle with AB = AC. Draw $AP \perp BC$ to show that $\angle B = \angle C$.*



To show : $\quad \angle B = \angle C$

Proof : $\quad$ In $\triangle ABP$ and $\triangle ACP$,

$\angle APB = \angle APC \qquad (= 90°) \quad [\because AP \perp BC]$

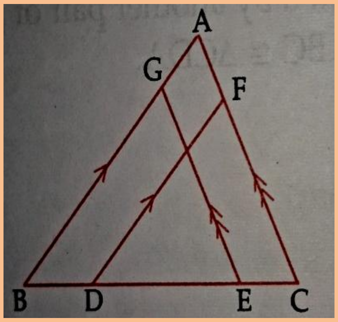
$AB = AC \qquad (Given)$

$AP = AP \qquad (common)$

$\therefore \quad \triangle ABP \cong \triangle ACP \qquad (RHS \text{ congruence rule})$

$\therefore \quad \angle B = \angle C \qquad (By\ \ C.P.C.T.) \qquad$ **Proved.**

Q. No. 13(c) ***In the adjoining figure,*** $BA \parallel DF$ ***and*** $CA \parallel EG$ ***and*** $BD = EC$.



Prove that (i) $BG = DF$ (ii) $EG = CF$

Proof: ***In*** $\triangle BEG$ ***and*** $\triangle DCF$,

$\angle B = \angle FDC$ [$\because BA \parallel DF$, ***corres.*** $\angle s$ ***are equal***]

$\angle BEG = \angle C$ [$\because CA \parallel EG$, ***corres.*** $\angle s$ ***are equal***]

$BD = EC$ (***Given***)

$BD + DE = EC + DE$ [***Adding both sides*** DE]

$BE = DC$

$\therefore$ $\triangle BEG \cong \triangle DCF$ (***ASA congruence rule***)

$\therefore$ $BG = DF$ (***By C.P.C.T.***)

And $EG = CF$ (***By C.P.C.T.***) ***Proved.***

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HOMEWORK

EXERCISE – 10.2

QUESTION NUMBERS: 5, 6, 8, 9, 11, 13 (a), (b) and 14.